Exercise 10

Find the closed form function for the following Taylor series:

$$f(x) = 1 + x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \cdots$$

Solution

$$\begin{aligned} f(x) &= 1 + x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \cdots \\ f(x) &= 1 + \frac{x^1}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \\ f(x) &= 1 + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} \end{aligned}$$

Therefore,

$$f(x) = 1 + \sin x.$$